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SUBOPTIMAL MULTICHANNEL DIGITAL FILTERS

Technical Report No. 2

SEISMIC ARRAY PROCESSING TECHNIQUES

Prepared by

Hal H. Bybee

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ABSTRACT

This report discusses a new method of generating time-domain filters to extract a signal from digitized multichannel noise. The new filter generation technique is based on the frequency-domain Wiener filter response and uses the mean-square-error of the whole filter set in the transformation back into the time domain. This new, computationally efficient technique was evaluated against a previously used technique, also based on the frequency-domain Wiener filter response. Two different sets of experimental noise data with power spectra specified at 65 frequencies were used in the filter evaluation, and the signal to be detected was assumed to have the same power spectrum as the noise to prevent frequency filtering. For the first noise sample and 37-point long filters, the previous technique gave 0.8 db more error than the optimum frequency domain filter and the new filtering technique gave 0.5 db more error. For the second noise sample, these respective filters had 1.2 db and 0.9 db more error than the optimum filter.

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SECTION I

INTRODUCTION

A new method of generating time-domain filters to extract a signal from digitized multichannel noise has been developed. These filters approximate optimal Wiener filters and for computational efficiency are designed in the frequency domain.

Two other techniques have been used to design multichannel filters. In all three techniques a noise sample, typical of the anticipated noise, is used to design a filter that will extract signals that match some predetermined signal model. The first previous technique was the optimal time-domain filter-design technique. In it the filter's mean-square-error was expressed in terms of the crosscorrelation matrix of the noise. Setting the partial derivatives of the error with respect to each filter-weighting coefficient equal to zero yields a set of simultaneous equations to be solved for these coefficients. If there were C channels of data and a P-point filter were desired, a set of C x P simultaneous equations would have to be solved.

In the second technique the filter was designed in the frequency domain. Even though this filter does not give as small a mean-square-error as the first filter, it is computationally much more efficient to obtain. In the frequency domain, the optimal Wiener filter frequency response was computed from the crosspower matrix of the noise sample. This step involved only solving one set of C simultaneous equations. In the second step of this technique, each channel of the desired frequency response, which was specified at many frequencies, was individually approximated in the least mean-square-error sense by a time-domain filter with only P points. This was accomplished by taking the inverse Fourier transform and evaluating it at the P desired time points. This system produces more than the optimal

Wiener error because the finite length impulse responses do not exactly fit the desired frequency responses required by the optimum filters. The new method uses this method as a standard for comparison.

The new filter generation technique is based on the frequency-domain Wiener filter response, as was the second filter. It is better than the first technique in that it uses much less computation than the first technique, and it is better than the second technique in that its mean-square-error for the data investigated is smaller. This experiment evaluated the new technique against the second technique since they both were relatively computationally efficient.

In the new technique, the mean-square-error of the whole filter set is used in the transformation back into the time domain, instead of just the error of one channel at a time (as in the second type of filter). The mean-square-error of the filter is expressed in terms of the crosspower matrix of the signal and the weighting coefficients of the filter. By factoring the crosspower matrix into the product of a lower triangular, a diagonal, and an upper triangular matrix before the error is computed, the error can be separated into one part that involves the coefficients and the coefficients of a second channel, one part that involves those two sets of coefficients and the coefficients of a third channel, and so on. A better filter set can then be generated by choosing the weighting coefficients of the first channel of the filter to minimize the first part of the mean-square-error, and then choosing the weighting coefficients of the second channel to minimize the second part of the error with the coefficients of the first channel substituted into this second part of the error, and so on.

This filter generation technique was programmed to evaluate its improvement in mean-square-error over that of the second filter technique discussed above. Two different sets of experimental noise data with

power spectra specified at 65 frequencies were used in the filter evaluation, and the signal to be detected was assumed to have the same power spectrum as the noise to prevent frequency filtering. For the first noise sample, a 37-point one-channel-at-a-time filter had 0.8 db more error than the optimum frequency domain filter and the new triangular-method filter using 37 points had 0.5 db more error. For the second noise sample these respective filters had 1.2 db and 0.9 db additional error. When the filters were restricted to 11 points, on the second noise sample the respective additional errors were 3.5 db and 2.1 db. For these two samples of seismic noise data this new technique which considers the interaction between channels gives a filter better fitting the frequency response needed to minimize the output mean-square-error over all channels.

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SECTION II

FILTER GENERATION TECHNIQUE

To illustrate the new triangular method filter technique, a 3-point filter will be analytically derived for a 2-channel signal. Given two digitized noise channels, the fast Fourier transform can be used to obtain an estimate of the spectral matrix of the noise

$$\begin{bmatrix} N_{11}(f) & N_{12}(f) \\ N_{21}(f) & N_{22}(f) \end{bmatrix} = N(f) \qquad (2\text{-}1)$$

N(f) will be positive definite and Hermitian. A model for the signal spectral matrix which will be Hermitian can be generated

$$\begin{bmatrix} S_{00}(f) & S_{01}(f) & S_{02}(f) \\ S_{10}(f) & S_{11}(f) & S_{12}(f) \\ S_{20}(f) & S_{21}(f) & S_{22}(f) \end{bmatrix} = S(f) \qquad (2\text{-}2)$$

In Equations 2-1 and 2-2, the subscripts 1 and 2 refer to the two channels and the subscript 0 refers to an origin point at which the signal estimate is desired.

The Wiener optimal filter H(f) for this signal and noise is given by[†]

$$\begin{bmatrix} S_{11}(f)+N_{11}(f) & S_{12}(f)+N_{12}(f) \\ S_{21}(f)+N_{21}(f) & S_{22}(f)+N_{22}(f) \end{bmatrix} \begin{bmatrix} H_1^*(f) \\ H_2^*(f) \end{bmatrix} = \begin{bmatrix} S_{10}(f) \\ S_{20}(f) \end{bmatrix} \tag{2-3}$$

where * denotes the complex conjugate.

The one-channel-at-a-time filter would have been realized from the Wiener response by minimizing the mean-square-error

$$\int_{-w}^{w} \left(F_j(f) - H_j(f)\right) \left(F_j^*(f) - H_j^*(f)\right) df \qquad j=1,2 \tag{2-4}$$

between the filter response and the Wiener response with respect to the filter coefficients. The filter response given in terms of its coefficients a_{jk} would be

$$F_j(f) = \sum_{k=1}^{3} a_{jk}\, e^{-i2\pi(k+c)\tau f} \qquad j=1,2 \tag{2-5}$$

In the above expressions, w and -w are the upper and lower limits of the signal bandwidth of data digitized with period τ. The zero point of the impulse response (Equation 2-5) of the filter is determined by the delay term c. The intergrand of the mean-square-error (expression 2-4) can be multiplied by a weighting factor R(f) to modify the impulse response of the filter, if desired.

The new triangular method filter is such a weighted form of expression 2-4. As derived by Burg,[†] the Wiener filter from Equation 2-3 has a mean-square-error between signal and output of

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[†] Burg, John P., 1964; Three-Dimensional Filtering with an array of Seismometers, Geophys., v. XXIX, n. 5, Oct. p. 693-713.

[†] Ibid.

$$\int_{-w}^{w} \left\{ S_{00}(f) - \begin{bmatrix} H_1(f) & H_2(f) \end{bmatrix} \begin{bmatrix} S_{10}(f) \\ S_{20}(f) \end{bmatrix} - \begin{bmatrix} S_{01}(f) & S_{02}(f) \end{bmatrix} \begin{bmatrix} H_1^*(f) \\ H_2^*(f) \end{bmatrix} \right.$$

$$\left. + \begin{bmatrix} H_1(f) & H_2(f) \end{bmatrix} \begin{bmatrix} S_{11}(f)+N_{11}(f) & S_{12}(f)+N_{12}(f) \\ S_{21}(f)+N_{21}(f) & S_{22}(f)+N_{22}(f) \end{bmatrix} \begin{bmatrix} H_1^*(f) \\ H_2^*(f) \end{bmatrix} \right\} df \qquad (2\text{-}6)$$

The error for any other filter G is the same as expression 2-6 with H replaced by G. Since the Wiener error is fixed, it can be subtracted from the error for G to give a simplified form of the error related to G. This excess mean-square-error derived from Equation 2-3 and expression 2-6 is

$$\int_{-w}^{w} \left\{ \begin{bmatrix} G_1(f)-H_1(f) & G_2(f)-H_2(f) \end{bmatrix} \begin{bmatrix} S_{11}(f)+N_{11}(f) & S_{12}(f)+N_{12}(f) \\ S_{21}(f)+N_{21}(f) & S_{22}(f)+N_{22}(f) \end{bmatrix} \begin{bmatrix} G_1^*(f)-H_1^*(f) \\ G_2^*(f)-H_2^*(f) \end{bmatrix} \right\} df \qquad (2\text{-}7)$$

This error is approximately minimized to give the new triangular filter by first factoring S+N into the product of a lower triangular, a diagonal, and an upper triangular matrix.

$$\begin{bmatrix} 1 & 0 \\ T_{12}^*(f) & 1 \end{bmatrix} \begin{bmatrix} D_{11}(f) & 0 \\ 0 & D_{22}(f) \end{bmatrix} \begin{bmatrix} 1 & T_{12}(f) \\ 0 & 1 \end{bmatrix} \qquad (2\text{-}8)$$

By substituting this expression for S+N, the error (expression 2-7) reduces to

$$\int_{-w}^{w} \left(G_1(f) - H_1(f) + T_{12}^*(f)\left(G_2(f) - H_2(f)\right)\right) D_{11}(f) \left(G_1^*(f) - H_1^*(f) + T_{12}(f)\left(G_2^*(f) - H_2^*(f)\right)\right) df \tag{2-9a}$$

$$+ \int_{-w}^{w} \left(G_2(f) - H_2(f)\right) D_{22}(f) \left(G_2^*(f) - H_2^*(f)\right) df \tag{2-9b}$$

G must be restricted to the form of Equation 2-5 to obtain a digital filter. It should be noted that expression 2-9b is independent of G_1 and is of the form of expression 2-4 with a weighting function of $D_{22}(f)$. For the triangular method filter, expression 2-9b is minimized with respect to the filter coefficients of G_2. This G_2 is then substituted into expression 2-9a which then takes on the form of expression 2-4 with a weighting function $D_{11}(f)$ and a modified Wiener response. Expression 2-9a is then minimized to evaluate G_1.

The only computational difference between this triangular method and the old weighted one-channel-at-a-time method is the computation of the modified Wiener response since the normal method of obtaining H from Equation 2-3 requires the triangular factorization of the S+N matrix. The computation of the modified Wiener response involves only the multiplication of a triangular matrix by a filter vector one row at a time as the filter vector is evaluated. This additional computation is very short in comparison to the amount of computation involved in the rest of the filter evaluation.

SECTION III

EVALUATION OF TECHNIQUE

The noise samples used in the computer evaluation of the triangular technique were both derived from actual seismic field measurements. For the measurements an array of 13 seismometers was used. The data coming from the seismometers were sampled, digitized, and recorded in the field. This set of data was returned to the lab where the power spectral matrix was computed for 65 equally-spaced frequencies from DC to the upper limit passed by the sampling unit.

The signal power spectra were assumed to have the same shape as the noise spectra, but the signal was given an infinite propagation velocity to model plane waves arriving normal or nearly normal to the earth's surface. A signal-to-noise ratio of 10 was used in the filter design.

SECTION IV

RESULTS AND CONCLUSIONS

Using the data discussed in Section III, three different length filters were designed and evaluated. Using the first noise sample, 37-point digital filters were designed with the output generated for the central point of the filter (i.e., the output was delayed half of the length of the filter). The The increased mean-square-error relative to the optimum frequency-domain filter was evaluated according to expression 2-7. The one-channel-at-a-time filter had 0.84 db mare error than the optimum Wiener filter, and the new triangular method filter had 0.53 db more error than the optimum filter. For the second noise sample for which the difference between spectra for all channels at each frequency was smaller than the first sample, the old-method filter had only 0.88 db excess error. Using the second noise sample, two more sets of shorter filters were evaluated. For the 21-point filter with output in the center, the old method yielded 2.40 db excess error and the new method yielded 1.24 db excess error. When an 11-point center output filter was designed, the excess errors were 3.50 db and 2.12 db, respectively.

The excess errors of both types of filters are, however, functions of the signal and noise power spectra so that for different noise environmants and signal characteristics, little can be said about the relative errors. If analytic expressions for noise and signal spectral matrices were known, analytic bounds on the errors could be derived from Equation 2-3 and expressions 2-6 and 2-7. For seismic applications, the results obtained from the noise samples of this report indicate that it should be well worth the investigator's effort to test the triangular method on oise and signals that he considers typical of his problem.

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